Fundamental constants and biology

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The issue of fine tuning

*Life is only possible for some values of the fundamental constants of physics* (Brandon Carter, Bernard Carr and Martin Rees)

John Barrow and Frank Tipler, *the Cosmological Anthropic Principle*

Martin Rees, *Our Cosmic Hambitat, Just Six Numbers*

Stephen Weinberg, the cosmological constant: 
Λ too large, no structures hence no life

Explanation: Multiverse?

Bdernad Carr (Ed): *Universe or Multiverse*
Fundamental constants of physics: parameters in the theory that cannot be reduced to other parameters

- Dependent on theoretical framework
- Should be dimensionless to be physically meaningful

Example: not $c$ or $\hbar$ or $G$

No agreement on number of constants, nor what they are

Comprehensive survey: Jean-Philippe Uzan
“Varying Constants, Gravitation and Cosmology”
The effect of changing the dimensionalities of space and time: astro-ph/9702052

The effect of changing the CMB fluctuation amplitude $Q \approx 10^5$: astro-ph/9709058


The effect on changing the dark matter density, dark energy density and CMB fluctuation amplitude: astro-ph/0511774

The effect of changing the masses of elementary particles: arXiv:0903.1024

Summary of effects: gr-qc/9704009. Annals of Physics 270, 1-51
Anthropic Limits: Tegmark’s diagram

Dimensions

Number of spatial dimensions

Number of time dimensions

UNPREDICTABLE (elliptic)

Tachyons only

TOO SIMPLE

We are here

UNPREDICTABLE (ultrahyperbolic)

UNSTABLE

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Fundamental constants and biology
S.M. Barr, A Khan (2007) “Anthropic tuning of the weak scale and of $m_u/m_d$ in two-Higgs-doublet models”
The fine-structure constant $\alpha$ is a fundamental physical constant characterizing the strength of the electromagnetic interaction between elementary charged particles. It is related to the elementary charge (the electromagnetic coupling constant) $e$, which characterizes the strength of the coupling of an elementary charged particle with the electromagnetic field, by the formula

$$\alpha = \frac{1}{4\pi \varepsilon_0} \frac{e^2}{\hbar c}$$

(1)

where

- $e$ is the elementary charge;
- $\hbar = h/2\pi$ is the reduced Planck constant;
- $c$ is the speed of light in vacuum;
- $\varepsilon_0$ is the electric constant or permittivity of free space.

Being a dimensionless quantity, it has the same numerical value in all systems of units. The currently accepted value of is

$$7.2973525698 \times 10^{-3} \iff \alpha^{-1} = 137.035999173.$$  

(2)
The Hoyle state

“Effects of the variation of fundamental constants on Pop III stellar evolution”
A variation of the fundamental constants is expected to affect the thermonuclear rates important for stellar nucleosynthesis. In particular, because of the very small resonant energies of Be8 and C12, the triple $\alpha$ process is extremely sensitive to any such variations. We derive limits on the variation of the magnitude of the nuclear interaction and model dependent limits on the variation of the fine structure constant based on the calculated oxygen and carbon abundances resulting from helium burning. The requirement that some C12 and O16 be present are the end of the helium burning phase allows for permille limits on the change of the nuclear interaction and limits of order $10^{-5}$ on the fine structure constant relevant at a cosmological redshift of $z \simeq 15 - 20$.
[arXiv:0911.2420]
Anthropic Limits: Tegmark’s diagram

Coupling constants

- Diproton disaster
- No non-relativistic atoms
- Carbon unstable
Cells,

Organic molecules: carbohydrates, lipids, proteins, nucleic acids (RNA, DNA)

functioning of living systems

What is possible is determined by possibility spaces

Evolutionary landscape: Waddington
Possibility spaces themselves exist as unchanging abstract (Platonic) spaces $\Omega_P$ limiting all possible structures and motions of physical systems. They are the same at all places and times. Our knowledge of them however is a representation of that space that is changing with time. That is, we represent $\Omega_P$ by some projection $\mathcal{P}(t) : \Omega_P \rightarrow E_P$ into a representation space $E_P$ where $\mathcal{P}(t)$ depends on the representation we use, and changes with time. This does not mean that physics itself, or the possibilities it allows, are changing: it is just that our knowledge of it is changing with time. Ontology (what possibilities exist, as a matter of fact) is entailed by the nature of $\Omega_P$. Epistemology (what we know about it) is determined by the projection $\mathcal{P}(t)$. The representation space $E_P$ will be represented via some coordinate system and set of units which can be altered without changing the nature of the entities being represented.
Biological Possibility spaces

Andreas Wagner: *The Arrival of the Fittest*: Platonic possibility spaces for micro biology
[Ard Louis talk]

Genotypes and phenotypes
- Gene transcription regulatory circuits
- Metabolic networks
- Protein genotype networks

Controlled by biological molecule shape: 3-dimensional folding, lock and key mechanism, promoters and effects

These are what control biological possibilities

How do they relate to the fundamental constants?
The functioning depends on intricate 3-d folding.
Very tight constraints in terms of bond lengths and angles
(Crick and Watson: Watson, *The Double Helix*)
The distance between adjacent sugars or phosphates in the DNA chain is 6 Angstroms. It must be between 5.5 and 6.5 Angstroms for it to work (p.20).
The doable problem
Viability bites

We can’t do DNA!

Nor even the water molecule ..

Which is a key molecule for life, particularly because of its dipole [Ball, *Life’s Matrix: A Biography of Water*; Ard Louis and Simon Conway Morris]

We’ll have to settle for the length of the bond in a Hydrogen molecule ... and hope that DNA will work out similarly!

- its a reasonable first step
Let us start with the standard Schrodinger equation. It takes the form

\[ i\hbar \partial_t \psi(r, t) = H\psi(r, t) \]  \( (3) \)

where \( H \) is the Hamiltonian.

Among the constants of the problem we choose \((m_e, \hbar, c)\) to construct our standard units. The units of time, length, and energy are thus given by

\[ \ell_a = \frac{\hbar}{m_e c}, \quad t_a = \frac{\hbar}{m_e c^2}, \quad E_a = m_e c^2. \]  \( (4) \)

For instance, the ionisation energy of hydrogen, given as

\[ -E_I = \frac{1}{2} m_e c^2 \alpha^2 = 13.60580 \text{ eV}, \]

in standard units, is

\[ -E_i = \frac{1}{2} \alpha^2 \]  \( (5) \)

in these atomic units.
The Schrödinger equation for a particle of mass $m$ is then

$$i\partial_{\tau} \psi(\rho, \tau) = -\left(\frac{1}{2} \frac{m_e}{m} \Delta + \frac{V}{m_e c^2}\right) \psi(\rho, \tau)$$  \hspace{1cm} (6)$$

in dimensionless form with potential $V$, where the space and time (dimensionless) coordinates are defined as

$$\tau = t/t_a, \hspace{1cm} \rho = \frac{r}{\ell_a}$$ \hspace{1cm} (7)$$

and $\Delta$ is Laplacian associated with $\rho$. 
At the lowest level, the Hamiltonian takes the form

\[ H_0 = \frac{P^2}{2m_e} - \frac{e^2}{4\pi\varepsilon_0 r} \] (8)

so that the dimensionless Schrödinger equation takes the form

\[ i\partial_\tau \psi(\rho, \tau) = -\left( \frac{1}{2} \frac{m_e}{m} \Delta + \frac{\alpha}{\rho} \right) \psi(\rho, \tau) \] (9)

Thus at the lowest level, rescaling \( \alpha \to A\alpha, \rho \to A\rho \) leaves second term invariant, but not the first (\( \nabla \) scales as \( \rho^{-2} \)). Hence the hydrogen atomic size will not just rescale with \( \alpha \).

**Hypothesis:** Changing \( \alpha \), chemistry will not work the same, with larger molecules. It will change conformation and so function.
Three combinations of constants:

The fine-structure constant:

\[ \alpha = \mu_0 e^2 c / 2\hbar \] (10)

and two others for future convenience:

\[ \beta = \hbar (2m_e c), \quad \gamma = c\hbar \] (11)

Because \( \mu_0 = 1/\epsilon_0 c^2 \), the fine-structure constant is also

\[ \alpha = e^2 / 2\epsilon_0 c\hbar = e^2 / 4\pi \epsilon_0 c\hbar \] (12)

We shall need

\[ e^2 / 4\pi \epsilon_0 = \alpha c\hbar = \alpha \gamma, \quad \hbar^2 (2m_e) = \beta \hbar c = \beta \gamma \] (13)

Note that the Bohr radius is

\[ a_0 = (4\pi \epsilon_0 \hbar^2) / (m_e e^2) = \hbar^2 / (\alpha c\hbar m_e) = 2\beta / \alpha \] (14)
The energies (MQM p 89; Z = 1) are

\[ E_n = -\frac{m_e e^4}{32\pi^2 \epsilon_0^2 \hbar^2} \cdot \frac{1}{n^2} = -\frac{m_e}{2\hbar^2} \left( \frac{e^2}{4\epsilon_0} \right)^2 \cdot \frac{1}{n^2} \]

so

\[ E_n = -\frac{\gamma \alpha^2}{4\beta} \cdot \frac{1}{n^2} \] (15)

The orbital of lowest energy (1s; n = 1, l = 0, ml = 0) is (MQM, p 88 and Y from p78)

\[ \psi_{1s}(r, \theta, \phi) = \frac{1}{\pi^{1/2}} \left( \frac{1}{a_0} \right)^{3/2} e^{-ra_0} = \frac{1}{\pi^{1/2}} \left( \frac{\alpha}{2\beta} \right)^{3/2} e^{-2\beta r \alpha} \]

The most probable distance of the electron from the nucleus is the Bohr radius, or \(2\beta/\alpha\).
But an atom is not a molecule. From MQM p265 in the LCAO (Linear Combination of Atomic Orbitals) approximation, for an internuclear separation R in $H_2^+$, with $S = R/a_0 = \alpha R/2\beta$:

$$E_+ - E_1s = \frac{j_0}{R} - \frac{j' + k'}{1 + S}, \quad j_0 = \frac{e^2}{4\epsilon_0} = \alpha \gamma$$

$$E_+ - E_1s = \frac{\alpha \gamma}{R} - \frac{j' + k'}{1 + S}$$

$$j' = \frac{j_0}{R} [1 - (1 + s)e^{-2s}] = \frac{\alpha \gamma}{R} \left[1 - \left(1 + \frac{1 + \alpha R}{2\beta}\right)\right] e^{-\alpha R/2\beta}$$

$$S = \left[1 + s + \frac{1}{3} s^2\right] e^{-s} = \left[1 + \frac{\alpha R}{2\beta} + \frac{1}{3} \left(\frac{\alpha R}{2\beta}\right)^2\right] e^{-\alpha R/2\beta}$$

$$k' = \frac{j_0}{a_0} [1 + s] e^{-s} = \frac{2\alpha^2}{\beta} \left[1 + \frac{\alpha R}{2\beta}\right] e^{-\alpha R/2\beta}.$$
Graph of energies of system as a function of radius. Binding will correspond to a minimum. Various values of $\alpha$ are shown: the blue one is the physical value of $\alpha$. Green and yellow are $\pm 6\%$ change in the binding radius.
Variation of energy with radius (on the x-axis marked 0 to 4) and change in fine structure constant on the y-axis. For each value of $\alpha$, the minimum energy gives the binding radius.
Finding the minimum: Graph of $dE/dr$ as a function of radius. Various values of $\alpha$ are shown: the blue one is the physical value of $\alpha$. Green is $\alpha$ reduced by 5.6%, and red is $\alpha$ increased by 6.4%. Both give $\pm 6\%$ change in the binding radius.
Different anthropic limits

What are the effects of varying $\alpha$ on the possibility spaces described by Wagner?

Which gives the tightest limits?

- Limits from biochemistry
- Limits from physics
- Limits from astrophysics

Which is the tightest?

Do the physics ones include the biology ones, or vice versa?

It may be that physics gives the tighter limits.

Does physics preview the existence of life?
It may be that the physics constraints may be tighter than those from life (compare with Tegmark figure).
Spin effects

Does electron spin affect biology? Do we have to take fine structure interactions into account?


The present status of the theory of electron spin effects in fundamental processes such as spin exchange and dipole-dipole interactions, electron transfer, triplet-triplet energy transfer, and annihilation intersystem crossing is reviewed. These effects form a basis for the understanding of the molecular mechanisms essential to chemical and biological reactions including photosynthesis and magnetic field influence, and for the creation of advanced organic magnets and catalysts, as well as the development of new methods of studying the structural and molecular dynamics of biological and non-biological objects.
Welcome to the Spin Chemistry Website
http://spinportal.chem.ox.ac.uk/

Broadly defined, Spin Chemistry deals with the effects of electron and nuclear spins in particular, and magnetic interactions in general, on the rates and yields of chemical reactions. It is manifested as spin polarization in EPR and NMR spectra and the magnetic field dependence of chemical processes. Applications include studies of the mechanisms and kinetics of free radical and biradical reactions in solution, the energetics of photosynthetic electron transfer reactions, and various magnetokinetic effects, including possible biological effects of extremely low frequency and radiofrequency electromagnetic fields, the mechanisms by which animals can sense the Earths magnetic field for orientation and navigation, and the possibility of manipulating radical lifetimes so as to control the outcome of their reactions.
“Electron spin changes during general anesthesia in Drosophila”
Turina, Skoulakisa, and Horsfield

One hundred sixty years after its discovery, the molecular mechanism of general anesthesia remains a notable mystery. A very wide range of agents ranging from the element xenon to steroids can act as general anesthetics on all animals from protozoa to man, suggesting that a basic cellular mechanism is involved. In this paper, we show that volatile general anesthetics cause large changes in electron spin in Drosophila fruit flies and that the spin responses are different in anesthesia-resistant mutants. We propose that anesthetics perturb electron currents in cells and describe electronic structure calculations on anesthetic-protein interactions that are consistent with this mechanism and account for hitherto unexplained features of general anesthetic pharmacology.
The fine and hyperfine structures can be determined using perturbation theory, that is by setting

\[ H = H_0 + W. \]  

(16)

The fine structure contains three contributions.

The spin-orbit interaction is described by

\[ W_{\text{S.O.}} = \frac{\alpha}{2m_e^2c^2} \frac{\hbar c g_e}{r^3} \frac{L \cdot S}{2}. \]  

(17)

where \( g_e \) is the electron gyromagnetic factor. It can be written in dimensionless form as

\[ \frac{W_{\text{S.O.}}}{m_e c^2} = \frac{\alpha g_e}{2 \rho^3} \frac{L \cdot S}{2 \hbar^2}. \]  

(18)

The second correction arises from the \((v/c)^2\)-relativistic terms and is of the form

\[ W_{\text{rel}} = -\frac{P^4}{8m_e^3c^2}. \]  

(19)
The third and last correction, known as the Darwin term, arises from the fact that in the Dirac equation the interaction between the electron and the Coulomb field is local. But, the non-relativist approximation leads to a non-local equation for the electron spinor that is sensitive to the field in a zone of order of the Compton wavelength centered on \( r \). It follows that

\[
W_D = \frac{\pi \hbar^2 q^2}{m_e^2 c^2} \delta(r). \tag{21}
\]

The average in an atomic state is of order

\[
\langle W_D \rangle = \frac{\pi \hbar^2 q^2}{(2m_e^2 c^2)|\psi(0)|^2} \sim m_e c^2 \alpha^4 \sim \alpha^2 H_0.
\]

Its dimensionless version is (by not forgetting that the Dirac distribution has a dimension)

\[
\frac{W_D}{m_e c^2} = \pi \alpha \delta(\rho). \tag{22}
\]
Including fine and hyperfine structure, the study of the hydrogen atom can be written as

\[ i\partial_\tau \psi(\rho, \tau) = - \left( \frac{1}{2} \frac{m_e}{m} \Delta + V \right) \psi(\rho, \tau) \] (23)

with the dimensionless potential, no longer a function of \((\alpha/\rho)\),

\[
V = \frac{\alpha}{\rho} + \left\{ \frac{\alpha}{2\rho^3} \frac{g_e}{2} \frac{L \cdot S}{\hbar} - \frac{\Delta^2}{8} + \pi \alpha \delta(\rho) \right\} + \frac{g_p}{2} \frac{m_e}{m_p} \left\{ - \frac{1}{2\rho^3} \frac{L \cdot \hbar}{\hbar} + \frac{2\pi}{3} \frac{g_e}{2} \frac{I \cdot S}{\hbar \cdot \hbar} \delta(\rho) \right. \\
\left. + \frac{1}{4\rho^3} \frac{g_e}{2} \left[ 3\left( \frac{S}{\hbar} \cdot n \right) \left( \frac{I}{\hbar} \cdot n \right) - \frac{I \cdot S}{\hbar \cdot \hbar} \right] \right\}.
\]

It is a function of 4 dimensionless constants: \(\alpha\), \(m_e/m_p\), \(g_e\) and \(g_p\).
Project:

Explore the effects of all the constants in the first and second terms of this Hamiltonian on biological function:

- Structure of DNA
- Protein Folding
- Wagner’s possibility spaces

See if they give tighter limits than the physics constraints, that is, the limits for atoms, nuclei, stars, and planets to exist
Fine tuning and constants

The multiverse view

Multiverse: Many variations of fundamental physics realised

Fundamental physics

Standard Model of particle physics

Organic molecules

Life

Fine tuning and the multiverse
Final Theory: Unique theory of fundamental physics realised

The problem with a Final Theory:

A Unique theory of fundamental physics would have the image of life written in
In summary,

- The anthropic literature considers in detail the effect of changes in the fundamental constants of physics on physical constraints in order that life can exist.

- These constants, e.g. $\alpha$, affect chemistry and hence affect biological function - giving true anthropic constraints.

- Exploring this relation is wide open and very interesting.

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