A Scale Free Universe

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Based on

No fifth force in a scale invariant universe
By Pedro G. Ferreira, Christopher T. Hill, Graham G. Ross.
arXiv:1612.03157 [gr-qc], to appear Phys. Rev .D.

Weyl Current, Scale-Invariant Inflation and Planck Scale Generation
By Pedro G. Ferreira, Christopher T. Hill, Graham G. Ross.
arXiv:1610.09243 [hep-th]. Phys.Rev. D95 (2017) no.4, 043507.

Scale-Independent Inflation and Hierarchy Generation
By Pedro G. Ferreira, Christopher T. Hill, Graham G. Ross.
arXiv:1603.05983 [hep-th]. Phys.Lett. B763 (2016) 174-178.

See also ...

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D. Blas, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D **84**, 044001 (2011)
J. Garcia-Bellido, J. Rubio, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D **84**, 123504 (2011).
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R. Jackiw and S. Y. Pi, Phys. Rev. D **91**, no. 6, 067501 (2015)
K. Kannike, G. Htsi, L. Pizza, A. Racioppi, M. Raidal, A. Salvio and A. Strumia, JHEP **1505**, 065 (2015)
G. K. Karananas and J. Rubio, Phys. Lett. B **761**, 223 (2016)
G. K. Karananas and M. Shaposhnikov, Phys. Rev. D **93**, no. 8, 084052 (2016)
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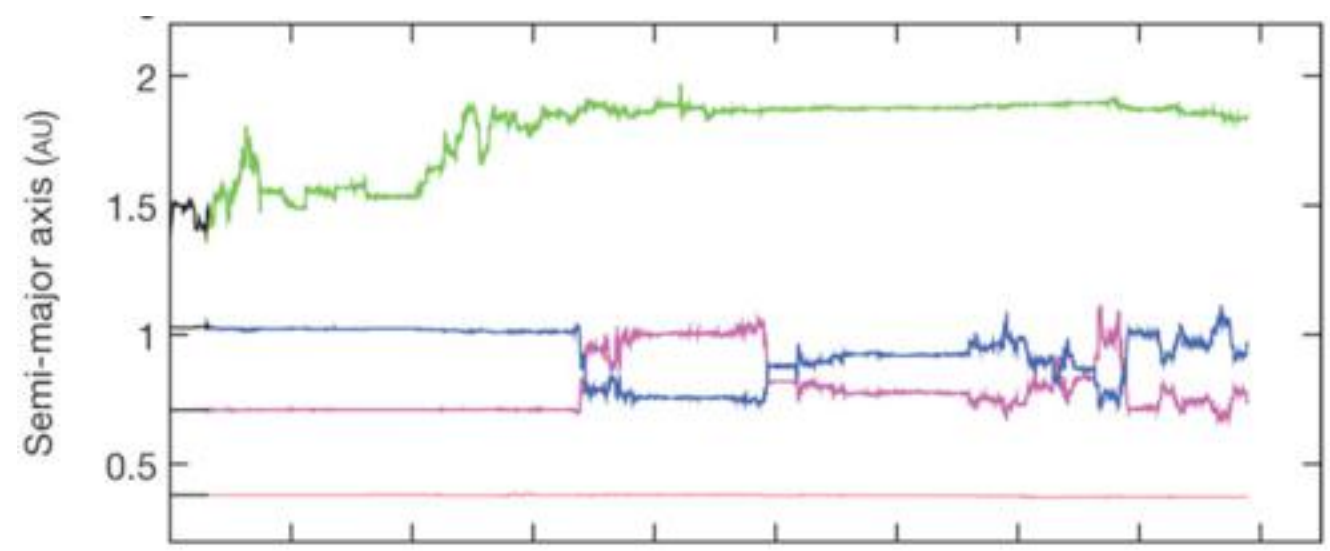
Constants aren't Constants

- Orbits of the planets



Laskar & Gastineu 2009

- Lepton masses

$$m\bar{\Psi}\Psi \rightarrow \lambda\phi\bar{\Psi}\Psi$$

Dirac’s Large Numbers

Dirac 1938

age of the Universe

$$\frac{t_U}{e^2/(m_e c^2)} \simeq \frac{e^2}{G m_p m_e}$$

atom light crossing time

ratio of em
to gravitational force

$$G \sim \frac{M_U}{R_U}$$

mass of visible universe

radius of visible universe

$$"\Box" G \sim \rho$$

Mach's Principle and a Relativistic Theory of Gravitation*

C. Brans† and R. H. Dicke
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received June 23, 1961)

The role of Mach's principle in physics is discussed in relation to the equivalence principle. The difficulties encountered in attempting to incorporate Mach's principle into general relativity are discussed. A modified relativistic theory of gravitation, apparently compatible with Mach's principle, is developed.

From Einstein-Hilbert to Brans-Dicke

From:

$$S_{EH} = -\int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + L_m\right]$$

where $M_{\rm Pl}^2 = \dfrac{1}{8\pi G}$

to:

$$S_{BD} = -\int d^4x\sqrt{-g}\left[-\frac{\alpha}{12}\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + L_m\right]$$

where $M_{\rm Pl}^2 = -\dfrac{\alpha}{6}\phi^2$

From Brans-Dicke to Einstein-Hilbert

Define $M^2 = -\frac{\alpha}{6}\phi^2$

to get

$$S_{BD} = -\int d^4x\sqrt{-g}\left[\frac{M^2}{2}R + \frac{\omega_{\rm BD}}{M^2}g^{\mu\nu}\partial_\mu M^2\partial_\nu M^2 + L_m\right]$$

Einstein-Hilbert recovered when $\omega_{\rm BD} \sim 1/\alpha \to \infty$

Mach's Principle and the Fifth Force

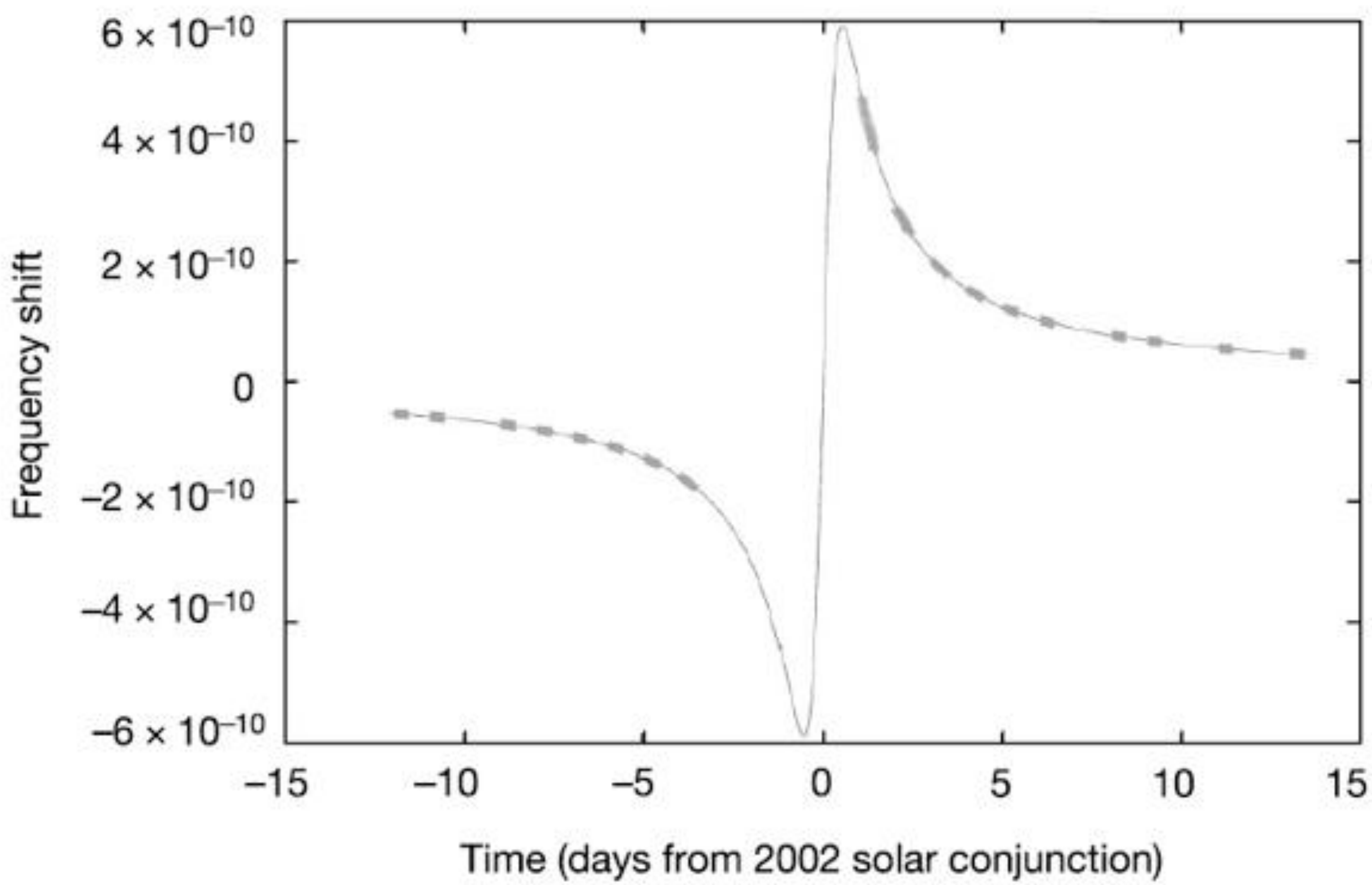
$$(1-\alpha)\left[\Box\phi + \frac{\nabla^\mu\phi\nabla_\mu\phi}{\phi}\right] = \frac{\rho+3P}{\phi}$$

Recall $"\Box" G \sim \rho$

"Newtonian Limit": $\phi = \phi_0 + \delta\phi$

$$(1-\alpha)\nabla^2\delta\phi \simeq -\frac{\rho}{\phi_0} \longrightarrow \delta\phi \sim \frac{M}{r}$$

Shapiro Time Delay



Bertotti et al 2003

Impact parameter

$$\text{Frequency shift} = -(1 \times 10^{-5}\text{s})(1+\gamma)\frac{d\ln b}{dt}$$

$$\gamma = \frac{2\alpha - 3}{4\alpha - 3} \qquad \gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$$

Generalized Scalar-Tensor Gravity

$$S = \int d^4x\sqrt{-g}\left\{\sum_{i=2}^{5}\mathscr{L}_i[\phi, g_{\mu\nu}] + \mathscr{L}_M[g_{\mu\nu}, \varphi]\right\}$$

$$\mathscr{L}_2 = K,$$
$$\mathscr{L}_3 = -G_3\Box\phi,$$
$$\mathscr{L}_4 = G_4R + G_{4X}\left\{(\Box\phi)^2 - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi\right\},$$
$$\mathscr{L}_5 = G_5G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left\{(\nabla\phi)^3 - 3\nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi\Box\phi + 2\nabla^\nu\nabla_\mu\phi\nabla^\alpha\nabla_\nu\phi\nabla^\mu\nabla_\alpha\phi\right\}.$$

Horndeski 1974

Deffayet et al 2009

where $X = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$

Scale (or Weyl) Invariance ...

Brans-Dicke with a quartic potential

$$S = -\int d^4x\sqrt{-g}\left[-\frac{\alpha}{12}\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \lambda\phi^4\right]$$

is invariant under scale transformations

$$g_{\alpha\beta} \to \Omega^2 g_{\alpha\beta}$$

$$\phi \to \phi/\Omega$$

$$x^\alpha \to \Omega x^\alpha$$

where Ω is a constant.

Scale (or Weyl) Invariance ...

The Noether current of this symmetry

$$K^\mu = (1-\alpha)\phi\nabla^\mu\phi \quad \text{satisfies} \quad \nabla_\mu K^\mu = 0$$

and is integrable

$$K^\mu = \nabla^\mu K$$

where we define the kernel

$$K = \frac{1}{2}(1-\alpha)\phi^2$$

... is broken spontaneously

Consider FRW metric $g_{\alpha\beta} = (-1, a^2\delta_{ij})$

Integrate the Noether current to get

$$\phi^2 = \phi_0^2 + c\int \frac{dt}{a^3}$$

Fixed point: ϕ_0

generates two mass scales:

$$\begin{cases} M^2 = -\dfrac{\alpha}{6}\phi_0^2 \\ H^2 = -\dfrac{2\lambda\phi_0^2}{\alpha} \end{cases}$$

Mass scales are generated spontaneously ...

... and are irrelevant. Only ratios matter.

$$\frac{H^2}{M^2} = 12\frac{\lambda}{\alpha^2}$$

Goldstone's Theorem

A spontaneously broken, continuous symmetry will lead to the existence of a massless boson which is derivatively coupled to any sources.

Goldstone 1961

In this case

symmetry ⟶ scale (or Weyl) invariance

boson ⟶ dilaton

but then

derivatively coupled ⟶ suppressed fifth force!

Is the Universe Scale Invariant?

First focus on classical scale invariance.

The field content:

- scalars → inflaton, Brans-Dicke field, Higgs, ...
- vectors → gauge fields
- tensors → metric

Multiscalars

$$S = -\int d^4x\sqrt{-g}\left[-\frac{1}{12}\sum_i^N \alpha_i \phi_i^2 R + \frac{1}{2}\sum_i^N \partial_\mu \phi_i \partial^\mu \phi_i - W(\vec{\phi})\right]$$

We can define a conserved current again $K^\mu = \nabla^\mu K$

such that $\nabla_\mu K^\mu = 0$ and $K = \frac{1}{2}\sum_{i=1}^N (1-\alpha_i)\phi_i^2$

Solve to find $K = K_0 + \int \frac{dt}{a^3}$

Scale symmetry spontaneously broken again!

Multiscalars

Conformal transformation with $\phi_i = e^{-\frac{\sigma}{f}}\hat{\phi}_i$ and $g_{\mu\nu} = e^{2\frac{\sigma}{f}}\hat{g}_{\mu\nu}$

σ is the dilaton and $\hat{\phi}_i$ satisfies $\bar{K} = \frac{1}{2}\sum_{i=1}^{N}(1-\alpha_i)\hat{\phi}_i^2 = f^2$

Transformed action:

$$S = -\int d^4x\sqrt{-\hat{g}}\left[-\frac{1}{12}\sum_i^N \alpha_i\hat{\phi}_i^2\hat{R} + \frac{1}{2}\sum_i^N \partial_\mu\hat{\phi}_i\partial^\mu\hat{\phi}_i + \frac{1}{f^2}\bar{K}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{f}\underbrace{\partial_\mu\sigma\partial^\mu\bar{K}}_{=0} - W(\vec{\hat{\phi}}) + \lambda_{\mathrm{L}}\mathcal{C}(\vec{\hat{\phi}})\right]$$

(i.e. no coupling $\longrightarrow$ no fifth force!)

Gauge Fields and Fermions

Gauge fields are conformally invariant: $A_\mu \to A_\mu$

Fermions transform as $\psi = e^{\frac{3\sigma}{2f}} \psi'$

and Higgs as $h = \hat{h} e^{-\frac{\sigma}{f}}$

But fermion action is invariant!

$$+\frac{i}{2}\bar{\psi}(\overrightarrow{\nabla\!\!\!/} - \overleftarrow{\nabla\!\!\!/})\psi - g'\bar{\psi}\psi h \longrightarrow +\frac{i}{2}\bar{\psi}'(\overrightarrow{\nabla\!\!\!/} - \overleftarrow{\nabla\!\!\!/})\psi' - g'\bar{\psi}'\psi'\hat{h}$$

_No_ coupling to dilaton $\longrightarrow$ _no_ fifth force.

No fifth force in a scale invariant universe

Cosmology of symmetry broken phase

Consider two scalar fields:

$$S = -\int d^4x\sqrt{-g}[\frac{1}{12}\alpha\phi^2 R + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$$

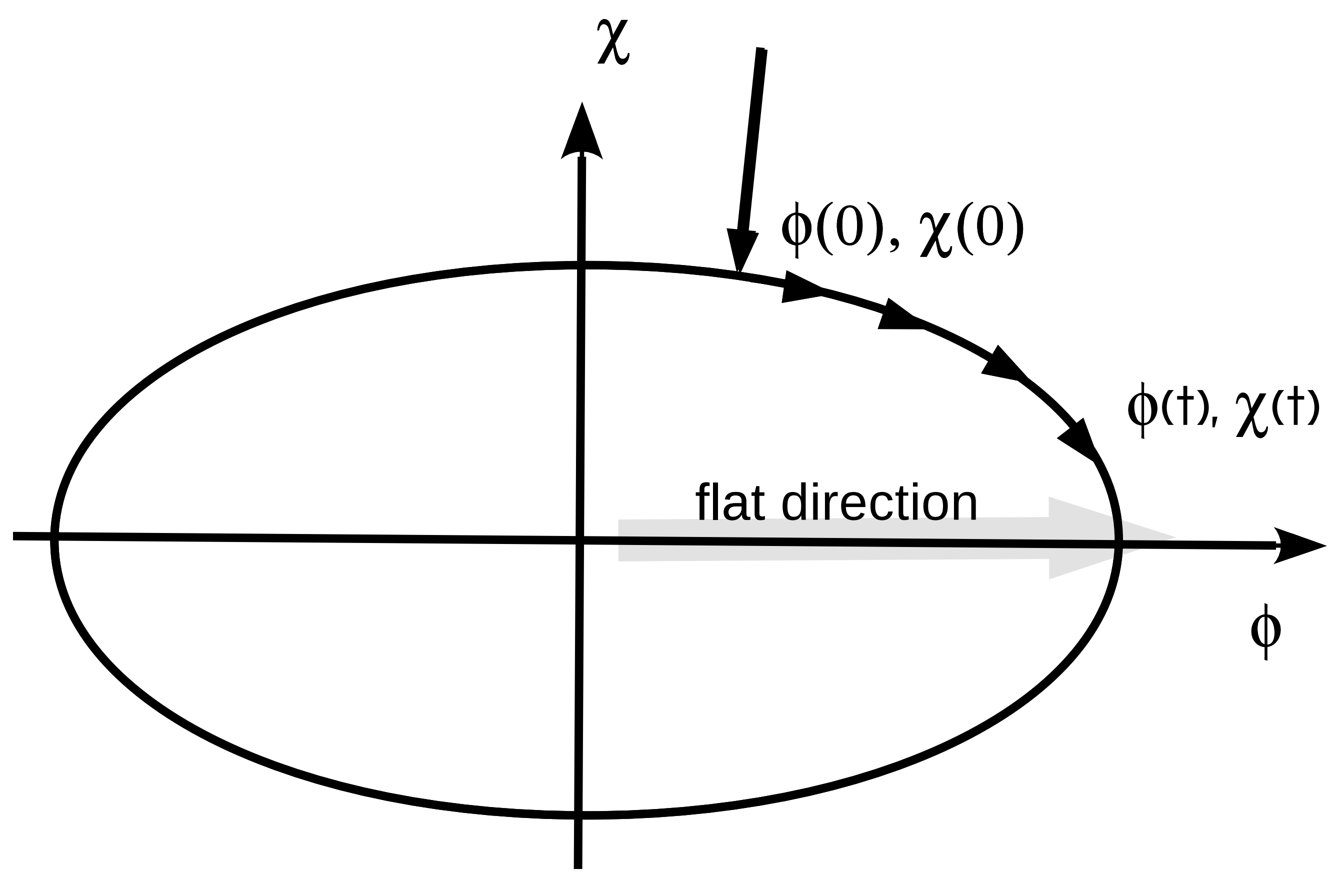
$$+\frac{1}{12}\beta\chi^2 R + \frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi + W(\phi,\chi)]$$

where $W(\phi,\chi) = \lambda\phi^4 + \xi\chi^4 + \delta\phi^2\chi^2$

Cosmology of symmetry broken phase

Symmetry broken phase

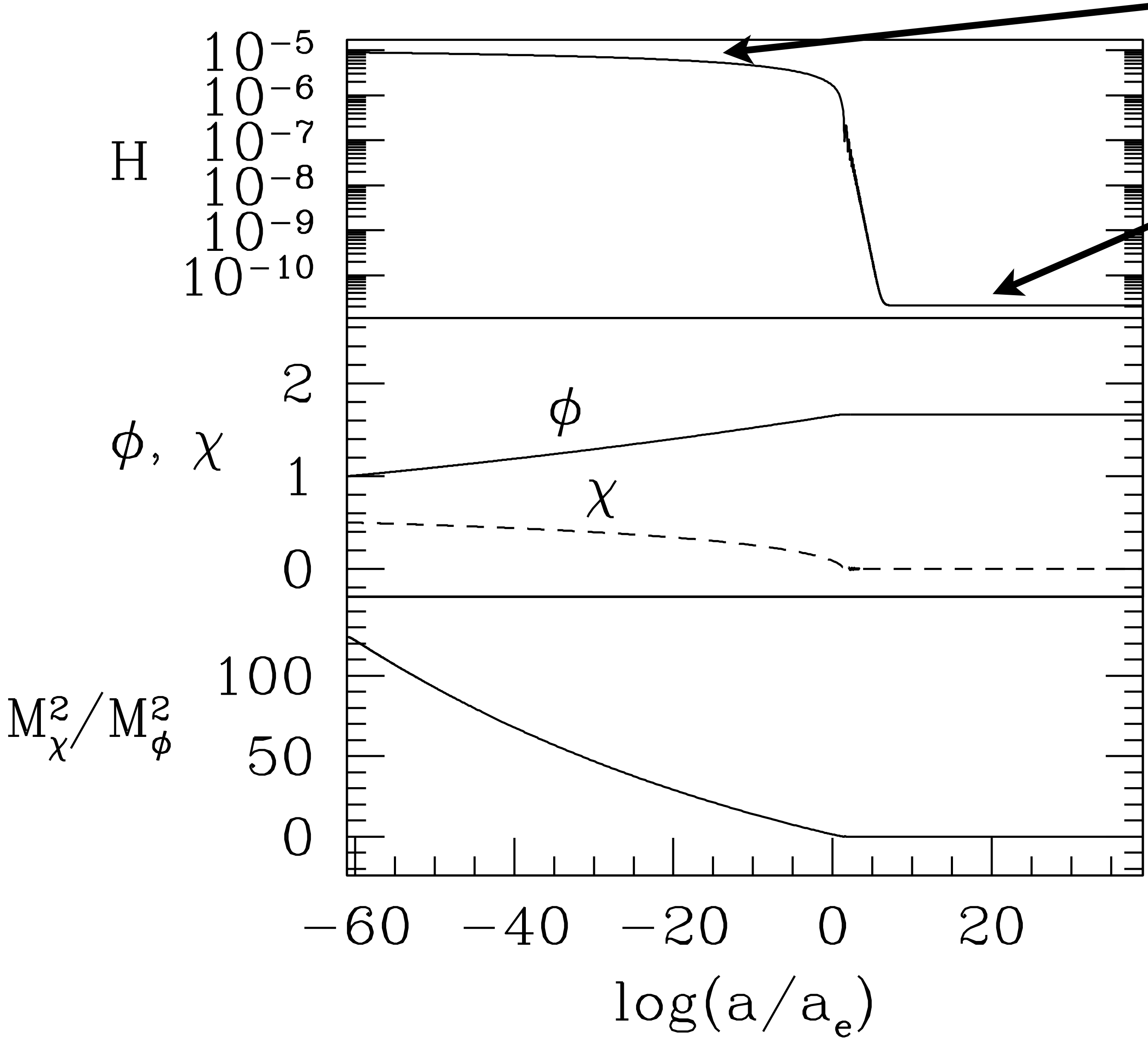
$$K = \frac{1}{2}(1-\alpha)\phi^2 + \frac{1}{2}(1-\beta)\chi^2 = \mathrm{constant}$$



Cosmology of symmetry broken phase

Inflation

Dark energy

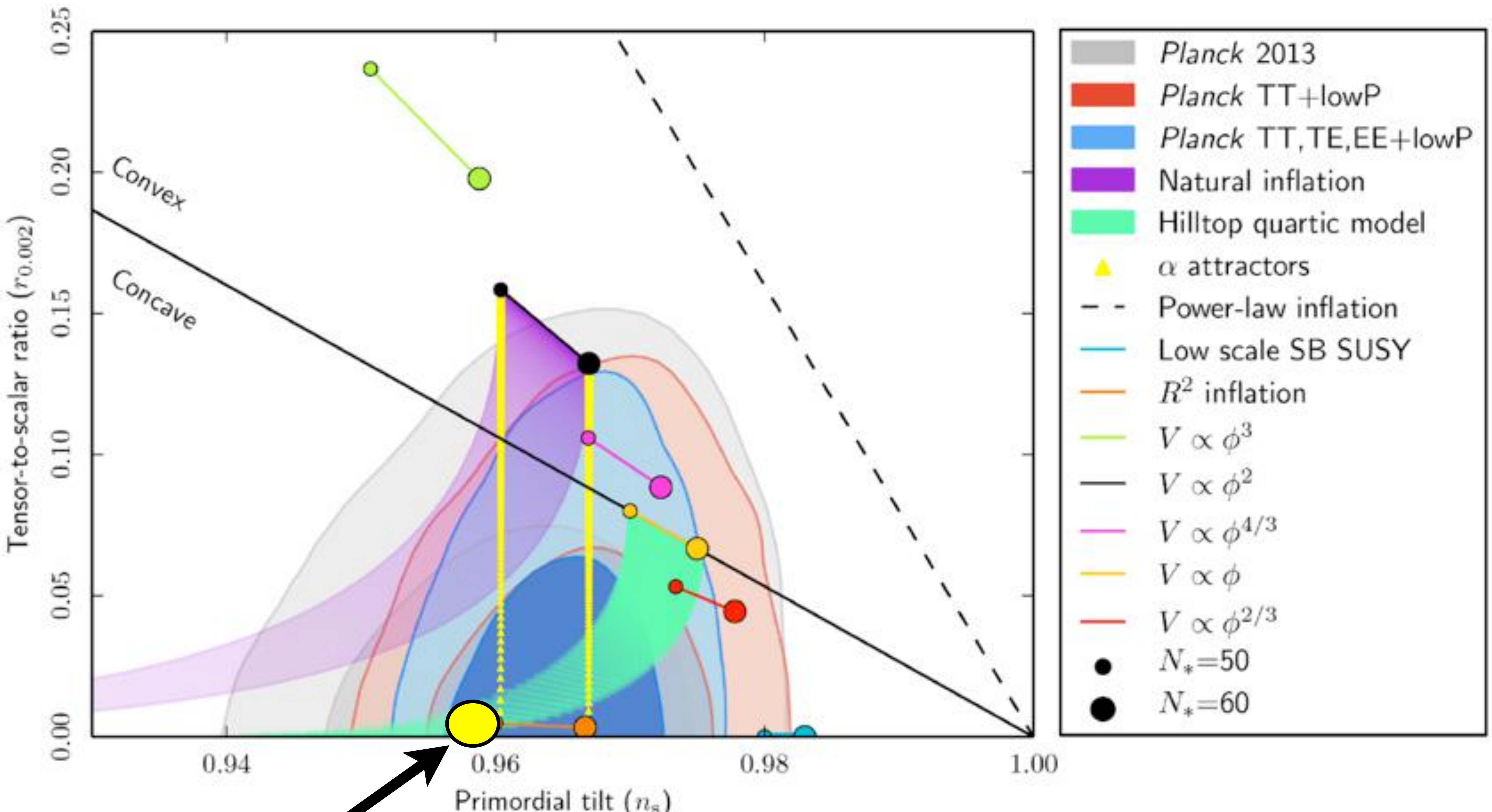


$$\frac{\langle\chi_0\rangle^2}{\langle\phi_0\rangle^2} = \frac{4\lambda\beta - 2\alpha\delta}{4\alpha\xi - 2\beta\delta}$$

$$M_\chi^2 = -\frac{\beta}{6}\chi^2 \qquad M_\phi^2 = -\frac{\alpha}{6}\phi^2$$

$$M_{\rm Pl}^2 = M_\chi^2 + M_\phi^2$$

Cosmology of symmetry broken phase



We are here

A viable classical cosmology

Quantum Mechanics

Scale symmetry of a theory is normally considered to be broken by quantum loops. However, this happens because at some stage in the renormalization procedure, we introduce explicit "external" mass scales into the theory by hand. These are mass scales that are not part of the defining action of the theory, and they lead to non-conservation of the scale current.

Scale Invariance ...

External mass scales enter through renormalization

field space. That is to say, we define the renormalized coupling constant by

M is external ⟶

$$\left.\frac{d^4V}{d\varphi_c{}^4}\right|_M = \lambda \,, \qquad (3.7)$$

where M is some number with the dimensions of a mass. We emphasize that M is completely arbitrary, just as is the corresponding quantity in the momentum-space analysis; different choices for M will lead to different definitions of the coupling constant, different parametrizations of the theory, but any nonzero M is as good as any other. Al-

Coleman-Weinberg 1973

... is anomalous.

Coleman-Weinberg potential

$$V(\phi) = \lambda\phi^4 \quad \longrightarrow \quad V(\phi) = \lambda\phi^4 + \frac{\beta_\lambda}{4}\phi^4 \ln(\phi/M)$$

breaks the conservation of scale current

$$\nabla_\mu K^\mu = 4V - \phi\frac{dV}{d\phi} = -\frac{1}{4}\beta_\lambda\phi^4 \neq 0$$

"Scale anomaly"

Scale Invariance ...

"Internal" mass scale enter through renormalization

Replace M by χ –

field space. That is to say, we define the renormalized coupling constant by

$$\left.\frac{d^4V}{d\varphi_c{}^4}\right|_{\chi} = \lambda \, , \tag{3.7}$$

where χ is some number with the dimensions of a mass. We emphasize that χ is completely arbitrary, just as is the corresponding quantity in the momentum-space analysis; different choices for χ will lead to different definitions of the coupling constant, different parametrizations of the theory, but any nonzero M is as good as any other. Al-

Coleman-Weinberg 1973

... can be preserved!

Coleman-Weinberg potential

$$V(\phi) = \lambda\phi^4 \longrightarrow V(\phi) = \lambda\phi^4 + \frac{\beta_\lambda}{4}\phi^4 \ln(\phi/\chi)$$

preserves conservation of scale current

$$\nabla_\mu K^\mu = 4V - \phi\frac{\partial V}{\partial \phi} - \chi\frac{\partial V}{\partial \chi} = 0$$

No "scale anomaly"!

Ellipse is deformed

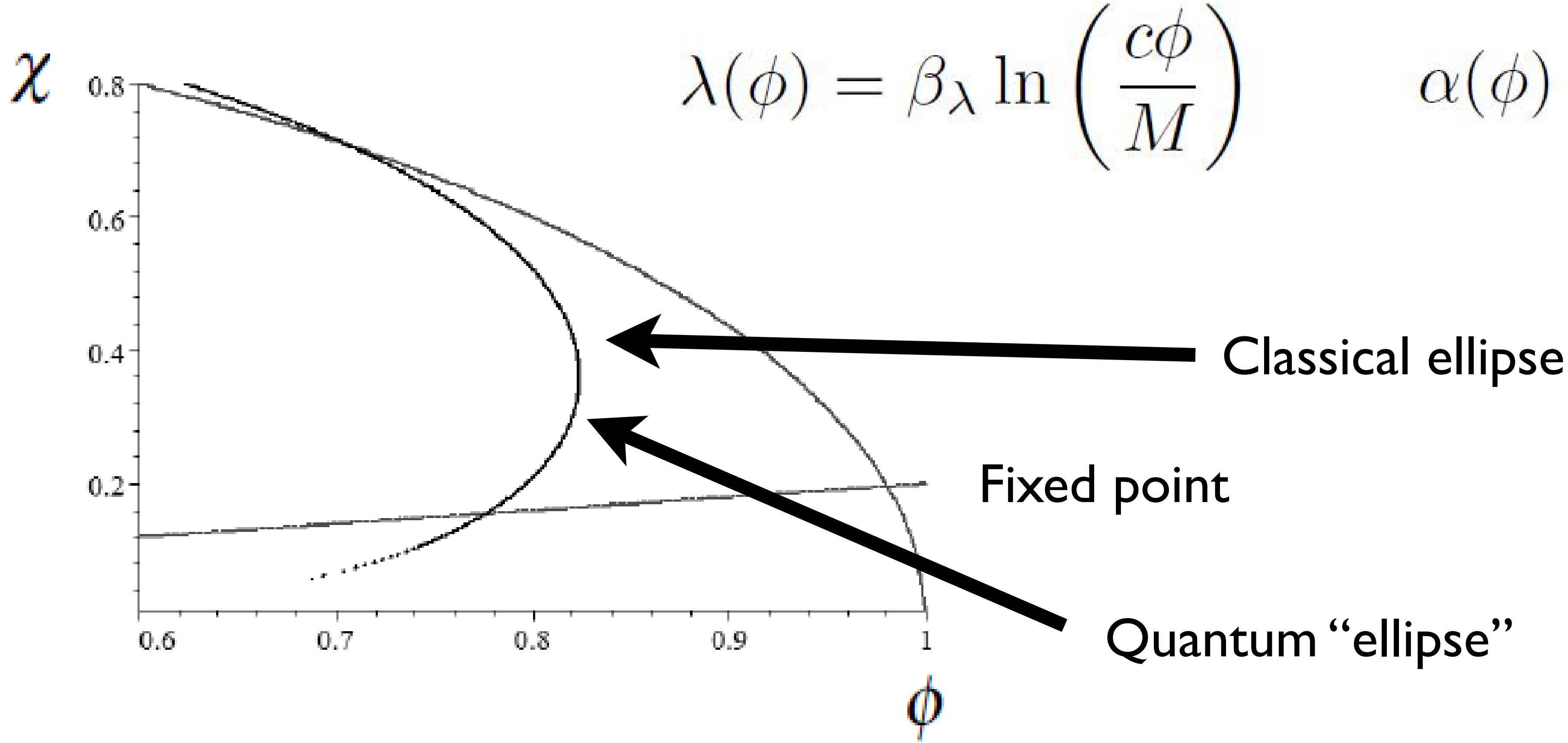
All dimensionless parameters run with RG

$$\phi \frac{\partial \lambda}{\partial \phi} = \beta_\lambda \quad \left(= \frac{9\lambda^2}{8\pi^2} \right)$$

$$\phi \frac{\partial \alpha}{\partial \phi} = \beta_\alpha = (\alpha - 1)\gamma_\alpha \qquad \left(\gamma_\alpha = \frac{3\lambda}{8\pi^2} \right)$$

One loop solution with internal renormalization

$$\lambda(\phi) = \beta_\lambda \ln\left(\frac{c\phi}{M}\right) \qquad \alpha(\phi) = 1 + (\alpha_0 - 1)\left(\frac{\phi}{M}\right)^{\gamma_\alpha}$$



A viable cosmology?

What next?

Does it have the usual (and devastating!) problems of inflation?

Are there any unique signatures from the early universe?

How does a scale invariant regularization prescription affect accelerator physics?

Does scale invariance help solve the old cosmological constant problem?

Are there dilaton signatures in strong gravity events such as the aLIGO black hole mergers?